BIEMBEDDINGS OF K_{13}

THOM SULANKE

Abstract. Using exhaustive searches all decompositions of K_{13} into two graphs which can be embedded on the torus or on the Klein bottle are obtained.

1. Introduction

A graph which can be embedded on a surface with Euler characteristic ε and which has n vertices has at most $3(n-\varepsilon)$ edges. So a graph which can be embedded on the torus or on the Klein bottle and which has 13 vertices has at most 39 edges. Suppose that the graph K_{13} which has 78 edges is the union of two graphs each of which can be embedded on a surface with Euler characteristic 0. Since this union would provide just enough edges for K_{13} each graph must triangulate the surface on which it can be embedded and each graph is the complement of the other.

Ringel [4], Beineke [1], and Jackson and Ringel [3] have provided various constructions which show that K_{6n+1} is the union of n copies of a graph which can be embedded on the torus. In particular for $n = 2$, these constructions provide a self-complementary graphs with 13 vertices which can be embedded on the torus.

Borodin and Mayer [2] each constructed a graph with 13 vertices which can be embedded on the torus and whose complement can be embedded on the Klein bottle.

Jackson and Ringel [3] have conjectured that there does not exist a graph with 13 vertices which can be embedded on the Klein bottle and whose complement can also be embedded on the Klein bottle.

2. Searching

Exhaustive computer searches were performed to find all graphs with 13 vertices for which both the graph and its complement can be embedded on surfaces with Euler characteristic 0. As pointed out above any such graph and its complement must be triangulations of surfaces with Euler characteristic 0.

The steps of one of these searches is outlined here. Suppose we want to find all the graphs with 13 vertices which can be embedded on the Klein bottle and whose complement can be embedded on the torus. (The computer times given are on a 2+ GHz processor with 1 Gbytes of memory.)

- Generate all the graphs with 13 vertices which triangulate the Klein bottle. Write each graph in canonical form to a list L_K. The triangulations are generated by a program which starts with irreducible triangulations and splits vertices until the desired number of vertices is reached. This list has

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21,535,942 entries of which 20,741,047 are unique. (Generate triangulations: 94 sec; put graphs in canonical form:142 sec)
- Sort L_K. (130 sec)
- Create a second list $L_{T'}$ of the complements of all the graphs with 13 vertices which triangulate the torus. Write each complement in canonical form to $L_{T'}$. This list has 8,778,329 entries of which 8,447,983 are unique. (Generate triangulations: 38 sec; find complements: 28 sec; put complements in canonical form: 56 sec)
- Sort $L_{T'}$. (44 sec)
- Compare the two lists L_K and $L_{T'}$ for common entries. (11 sec)

3. Figures

The figures of triangulations shown here consist of four parts. On the left of each figure is a polygon representing the triangulation. The rotation is given in the upper right of each figure. Below the rotation is a list of generators for the automorphism group of the triangulation. If the automorphism group is trivial then no generators are shown. In the lower right of each figure is the *degree sequence* of the embedded graph which is a list of the degrees of the vertices in non-decreasing order.

4. Torus - torus biembeddings

An exhaustive search found 22 graphs with 13 vertices which can be embedded on the torus and whose complements can also be embedded on the torus. Of these 22 graphs 6 are self-complementary and the other 16 graphs form 8 pairs of non-isomorphic complementary graphs. One of these graphs can be embedded on the torus in two different ways. Triangulations on the torus using these 22 graphs are shown in Figs. 5, 7, 9, 11, 13, 15, 17, 18, 19, 20, and 23 through 34. That these 22 graphs are unique can be seen from their degree sequences except for Figs. 33 and 34. But these two are not isomorphic because in Figure 33 vertex f, the only vertex with degree 4, has four neighbors of degree 9 while in Figure 34 vertex h, the only vertex in this graph with degree 4, has two neighbors of degree 9.

Figures 5 through 34 show the 15 ways the edges of K_{13} can be decomposed into two subgraphs both of which can be embedded on the torus.

These decompositions are denoted as tt1 through tt15. Decompositions tt1 through tt6 consist of six self-complementary graphs and their isomorphic complements. Decomposition tt1 is equivalent to the one generated by the method described by Ringel in [4]. Decomposition tt2 is equivalent to the one generated by Beineke in [1]. Jackson and Ringel [3] also produce this decomposition using the current graph reproduced here in Figure 1.

If we modify in turn the direction of rotation of each of the vertices in Figure 1 we get the current graphs in Figs. 2, 3, and 4. These current graphs produce tt3, tt4, and tt1.

The pairs of graphs used in tt8 and tt9 are the same. The triangulations in Figs. 19 and 21 are the same. The graphs in Figs. 20 and 22 are the same but they are embedded differently. To see this consider the two faces akd and blc in each triangulation which are the only faces with vertices of degree $\{3, 9, 9\}$. In Figure 20 both of these faces are adjacent to faces with vertices of degree $\{3, 6, 9\}$, $\{3, 6, 9\}$, and $\{6, 9, 9\}$ while in Figure 22 these faces are adjacent to faces with vertices of

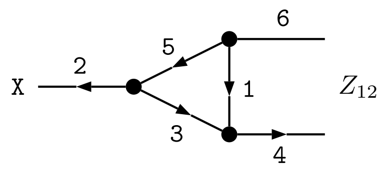

Figure 1. Current graph for tt2

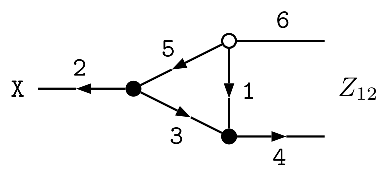

Figure 2. Current graph for tt3

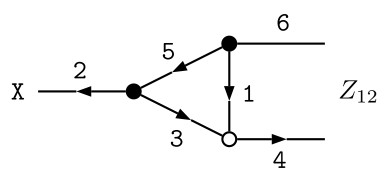

Figure 3. Current graph for tt4

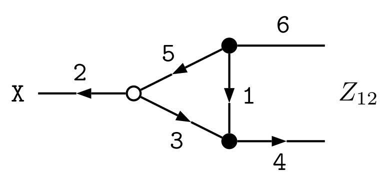

Figure 4. Current graph for tt1

degree $\{3,6,9\}$, $\{3,6,9\}$, and $\{5,9,9\}$. So the two triangulations in Figs. 20 and 22 are not isomorphic.

5. Torus - Klein bottle biembeddings

An exhaustive search found 18 graphs with 13 vertices which can be embedded on the torus and whose complements can be embedded on the Klein bottle. Triangulations of these graphs on the torus and triangulations of their respective complements on the Klein bottle are shown in Figs. 35 through 70.

All 18 graphs are unique. This can be seen from the degree sequences with the exceptions of two pairs of graphs. Graphs tk4t and tk7t in Figs. 41 and 47 have the same degree sequence, however, the one vertex of degree 5, l, in Figure 41 is adjacent to a vertex of degree 3 while the one vertex of degree 5, k, in Figure 47 is not adjacent to a vertex of degree 3. Graphs tk11t and tk16t in Figs. 55 and 65 also have the same degree sequence. Consider the set of vertices in each of these two graphs which are adjacent to two vertices of degree 3. In tk11t these vertices are $\{j,k,l\}$ while in tk16t they are $\{j,l,m\}$. If these two graphs were isomorphic then the isomorphism must map $\{j,k,l\}$ of tk11t onto $\{j,l,m\}$ of tk16t. But in tk11t b which has degree 3 is adjacent to each element of $\{j, k, l\}$ while in tk16t each vertex of degree 3 is adjacent to only two elements of $\{j, l, m\}$.

6. Klein bottle - Klein bottle biembeddings

An exhaustive search found no graphs with 13 vertices which can be embedded on the Klein bottle and whose complement can also be embedded on the Klein bottle. This confirms the conjecture of Jackson and Ringel [3].

References

1. L. W. Beineke, *Minimal decompositions of complete graphs into subgraphs with embeddability properties*, Canad. J. Math. **21** (1969), 992–1000. MR 39 #6783
2. Oleg V. Borodin and Jean Mayer, *Decomposition of K_{13} into a torus graph and a graph imbedded in the Klein bottle*, Discrete Math. **102** (1992), no. 1, 97–98. MR 1 168 137
3. Brad Jackson and Gerhard Ringel, *Variations on Ringel's earth-moon problem*, Discrete Math. **211** (2000), no. 1-3, 233–242. MR **2000h:**05167
4. Gerhard Ringel, *Die toroidale Dicke des vollständigen Graphen*, Math. Z. **87** (1965), 19–26. MR 30 #2489

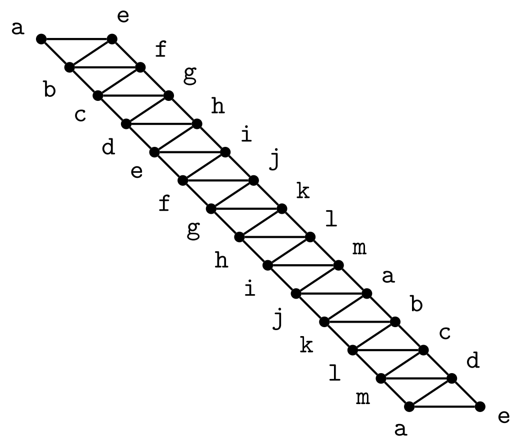

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FIGURE 5. tt1a

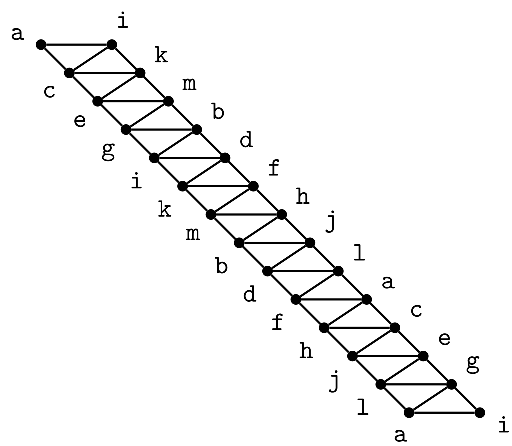

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FIGURE 6. tt1b

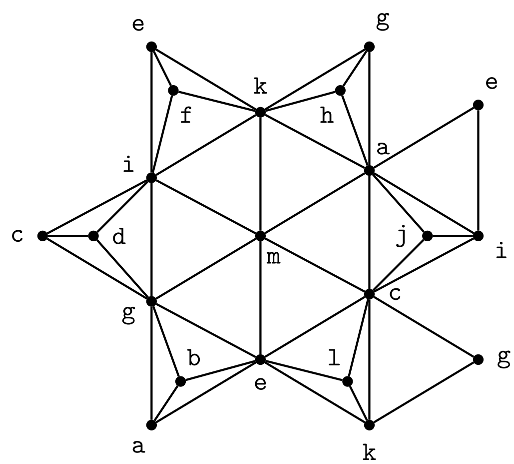

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FIGURE 7. tt2a

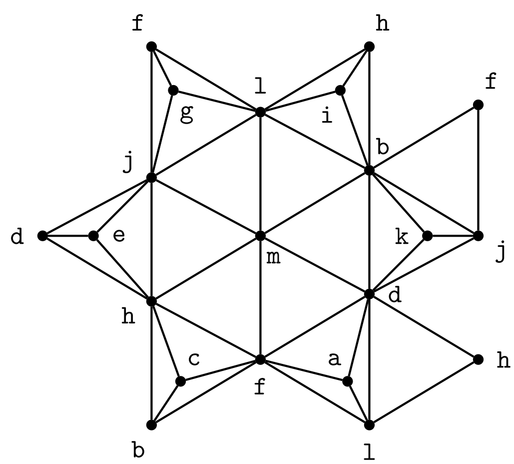

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FIGURE 8. tt2b

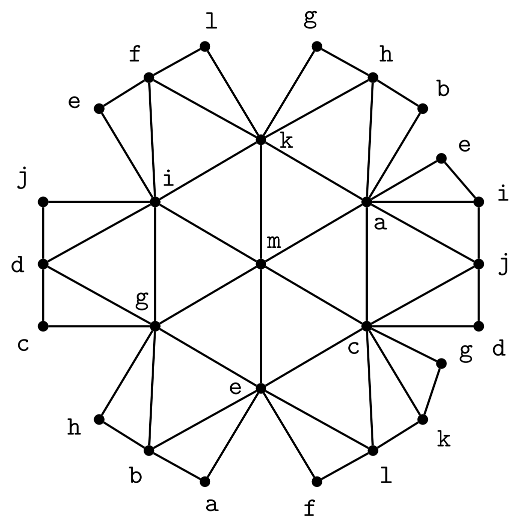

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Figure 9. tt3a

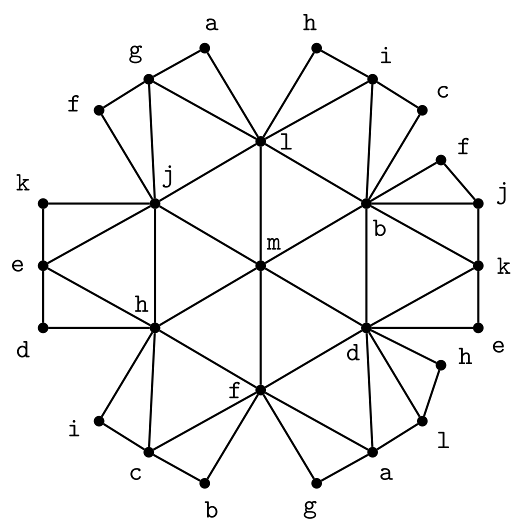

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Figure 10. tt3b

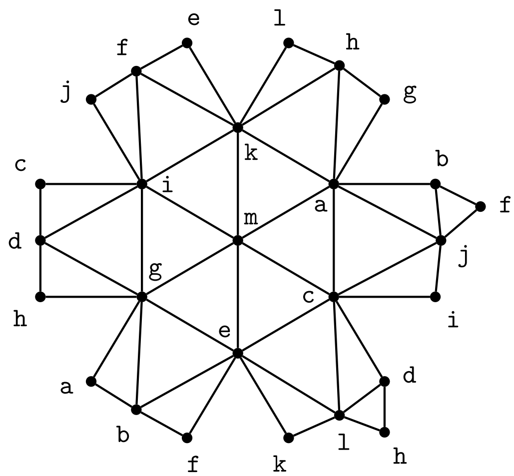


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Figure 11. tt4a

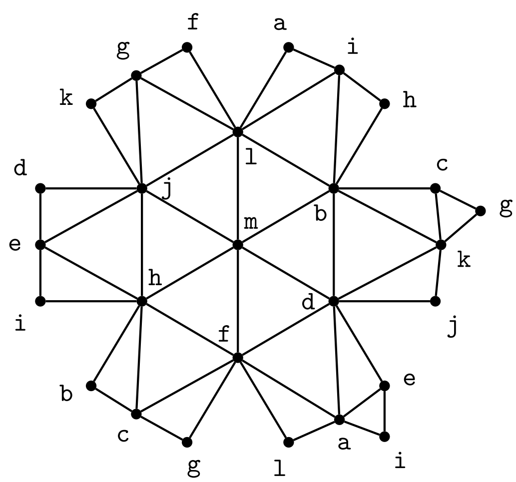


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Figure 12. tt4b

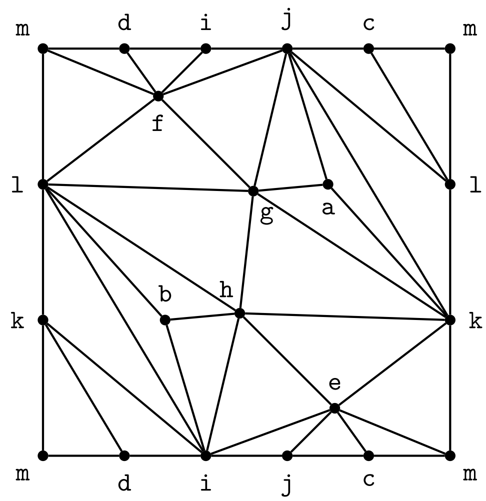

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FIGURE 13. tt5a

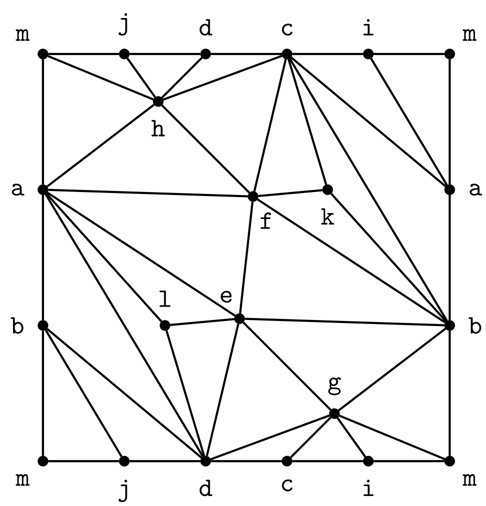

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FIGURE 14. tt5b

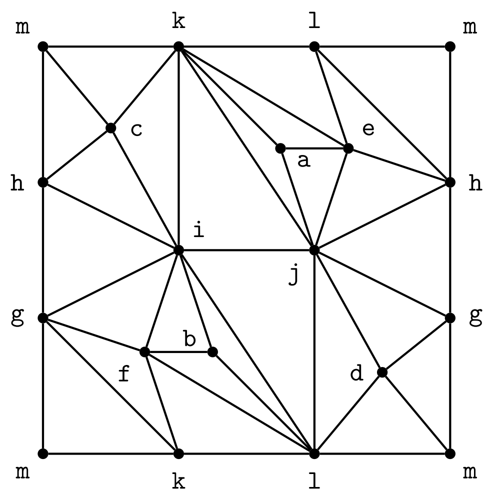

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Figure 15. tt6a

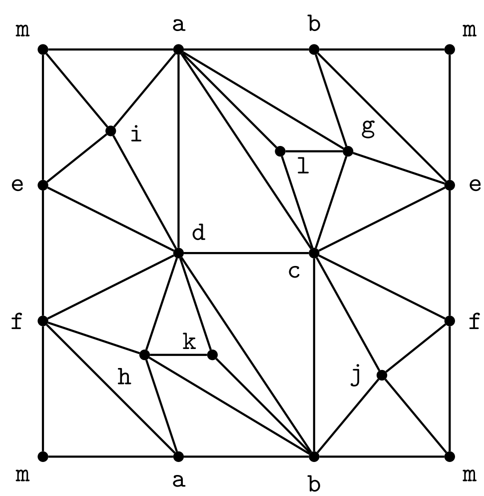

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Figure 16. tt6b

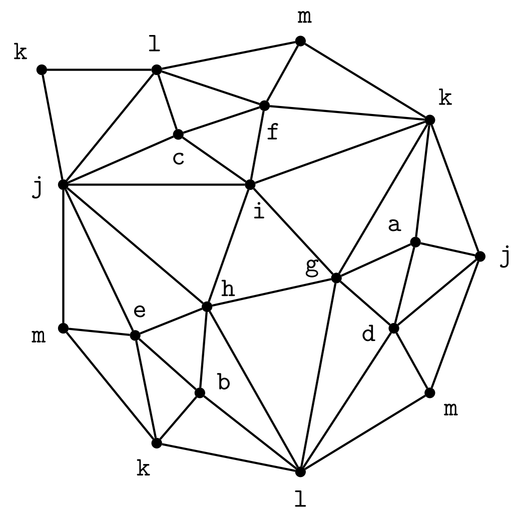

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FIGURE 17. tt7a

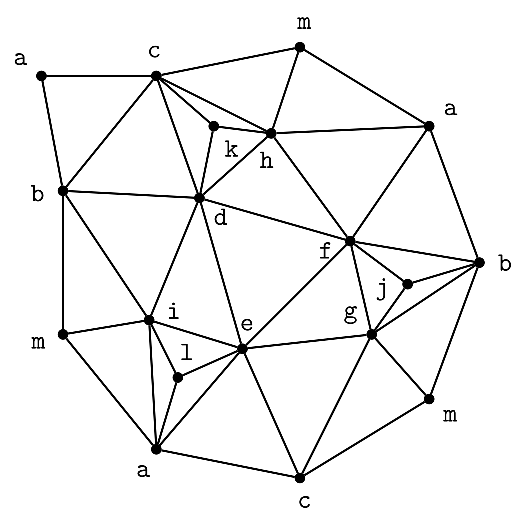

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FIGURE 18. tt7b

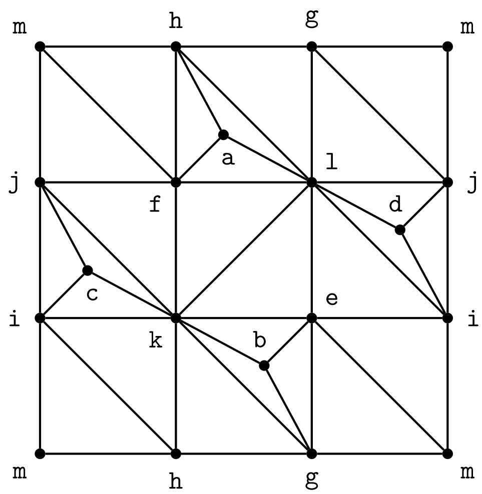

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FIGURE 19. tt8a

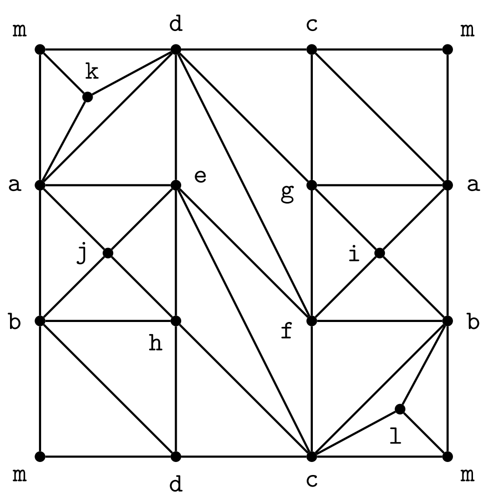

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FIGURE 20. tt8b

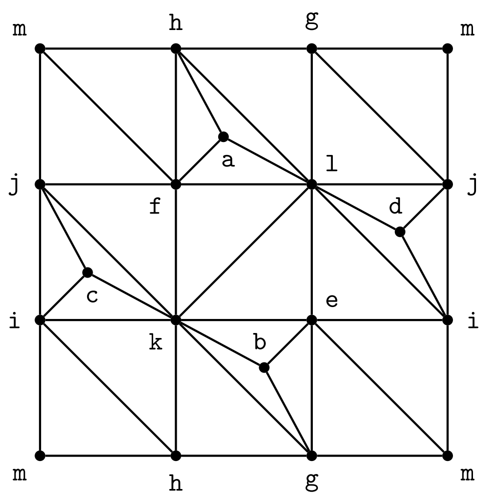


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FIGURE 21. tt9a

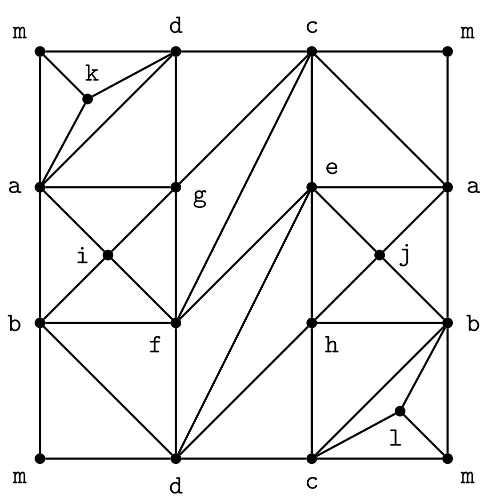


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FIGURE 22. tt9b

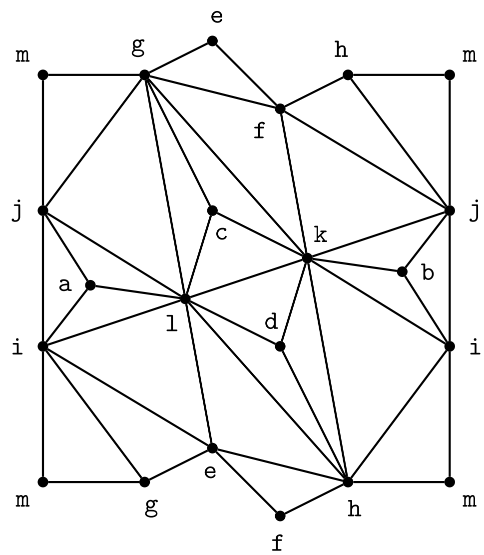

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FIGURE 23. tt10a

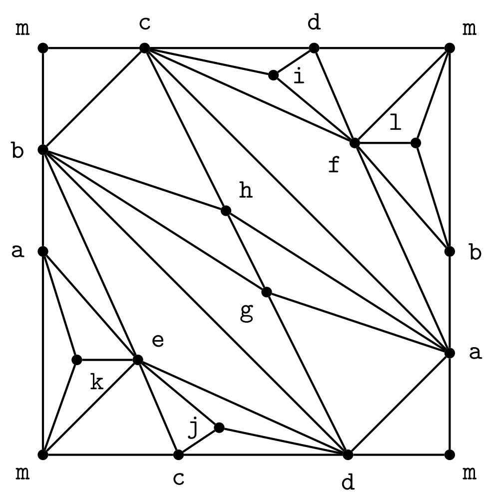

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FIGURE 24. tt10b

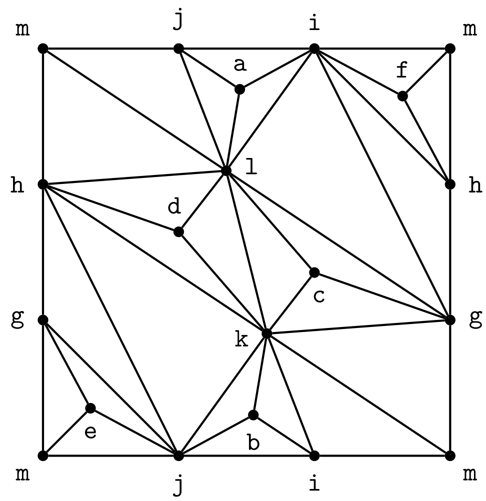

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FIGURE 25. tt11a

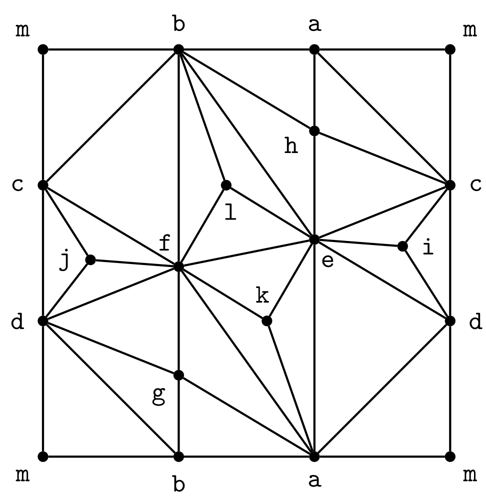

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FIGURE 26. tt11b

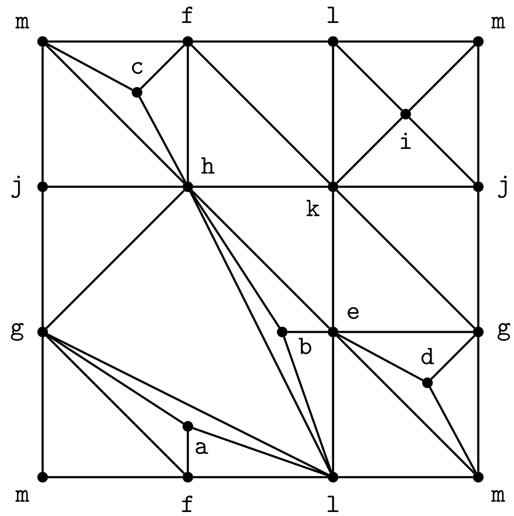

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FIGURE 27. tt12a

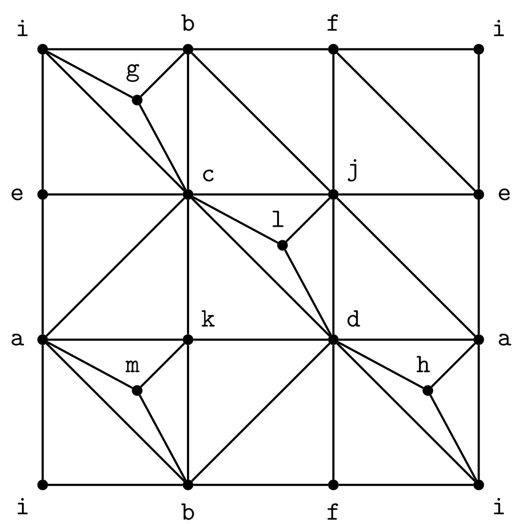

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FIGURE 28. tt12b

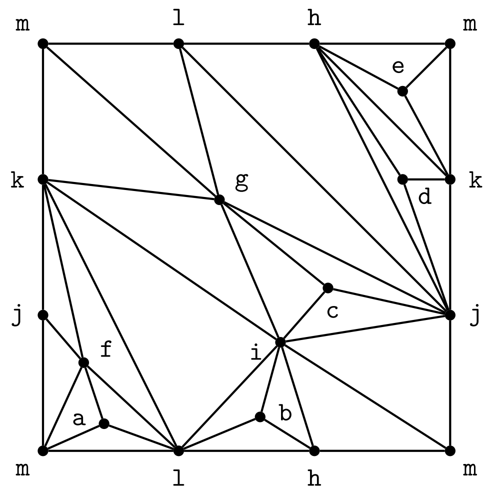

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FIGURE 29. tt13a

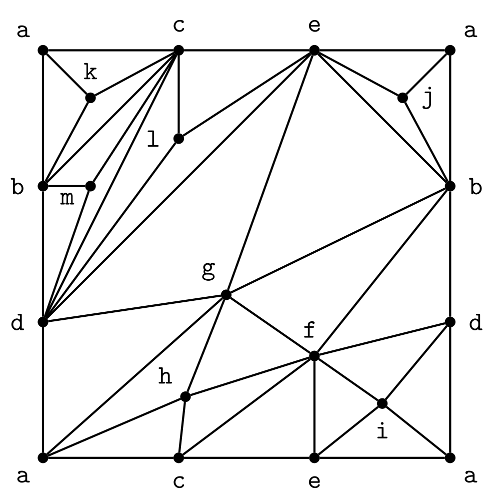

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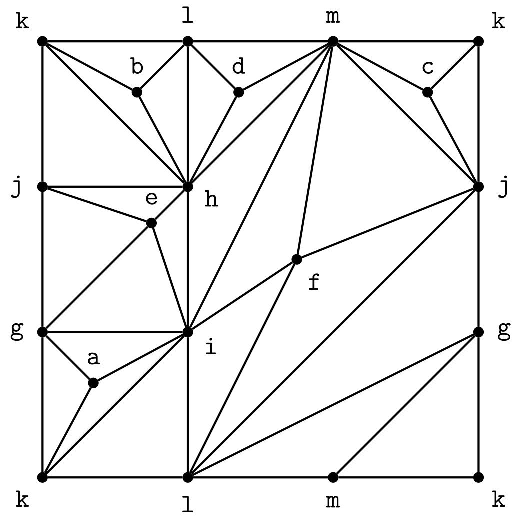
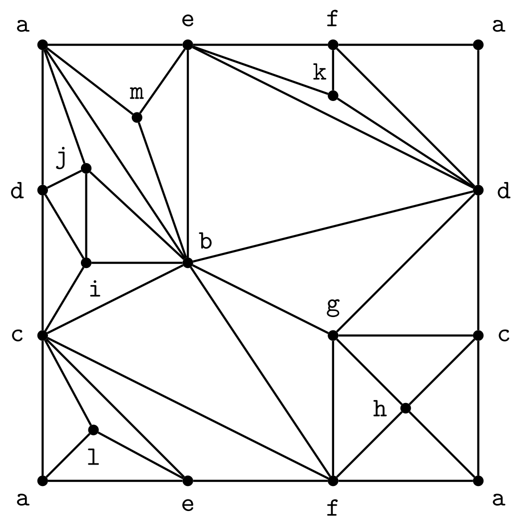
FIGURE 30. tt13b



a kgi
b klh
c kjm
d lmh
e gjhi
f imjl
g akmljei
h kbldmiej
i agehmflk
j kheglfmc
k ailbhjcmg
l kifjgmdhb
m kcjfihdlg

3333447888999

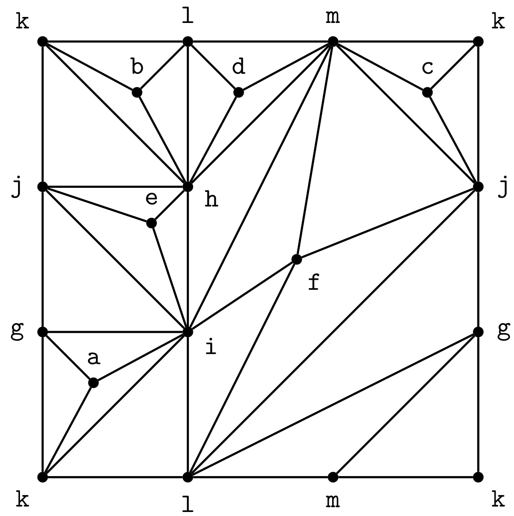
Figure 31. tt14a



a lembjdfhc
b amedgfcij
c ahgdibfel
d ajicgbekf
e alcfkdbm
f adkecbgh
g bdchf
h afgc
i bcdj
j abid
k efd
l ace
m aeb

3334445889999

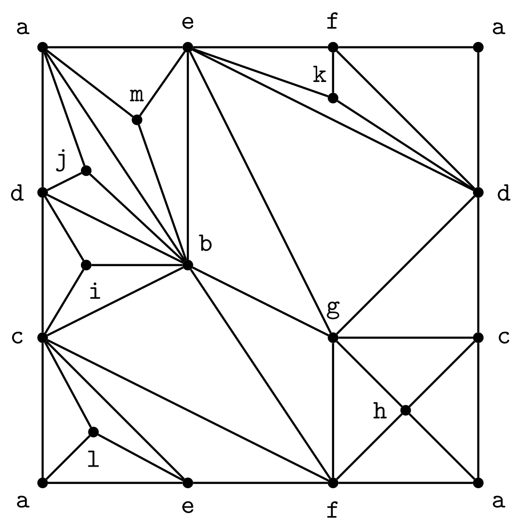
Figure 32. tt14b



a kgi
b klh
c kjm
d lmh
e jhi
f imjl
g akmlji
h kbldmiej
i agjehmflk
j kheiglfmc
k ailbhjcmg
l kifjgmdhb
m kcjfihdlg

3333346899999

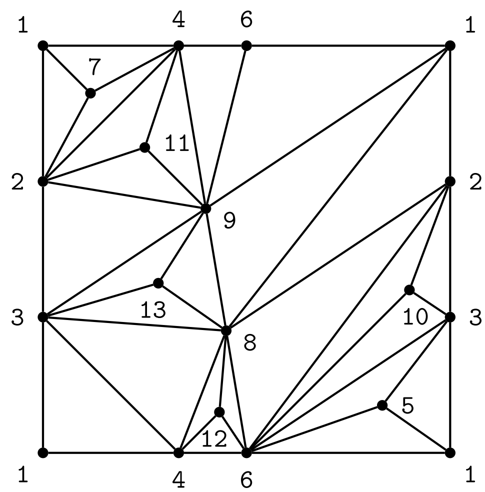
FIGURE 33. tt15a



a lembjdfhc
b amegfcidj
c ahgdibfel
d ajbicgekf
e alcfkdgbm
f adkecbgh
g bedchf
h afgc
i bcd
j abd
k efd
l ace
m aeb

3333346899999

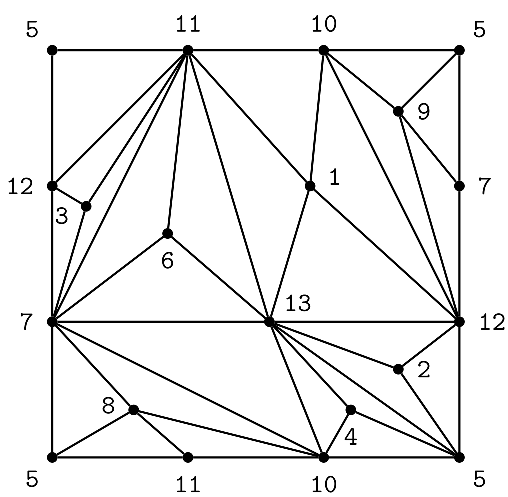
FIGURE 34. tt15b



a dcefihbg
b dgahfjcik
c dhmibjfea
d bkiflhcag
e acf
f diaecjbhl
g dab
h dlfbaimc
i dkbcmhaf
j bfc
k dbi
l dfh
m hic

3333338889999

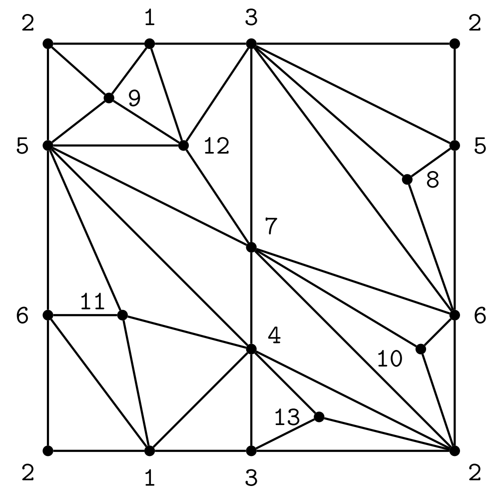
Figure 35. tk1t Torus of O.V. Bordin



a mljk
b mel
c lkg
d mje
e dmblkhgij
f mkg
g mjheilckf
h jkeg
i jegl
j dmghkalie
k majhelcgf
l mbekcgija
m djgfkalbe

3333444999999

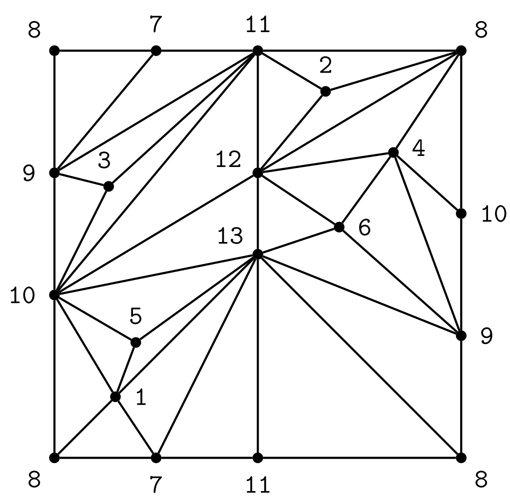
Figure 36. tk1k Klein bottle of O.V. Bordin



a libfkdc
b ecmdgjfai
c ehfgladmb
d egbmcak
e lgdkfhcbi
f ekabjgch
g elcfjbd
h efc
i ebal
j fbg
k edaf
l eiacg
m dbc

3334457778999

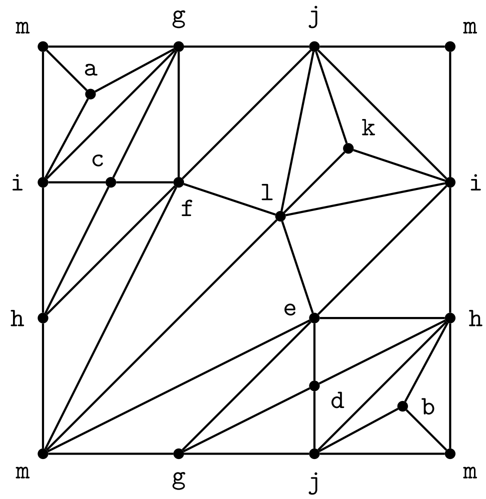
FIGURE 37. tk2t Torus of J. Mayer



a emghj
b lhk
c jik
d jiflh
e maj
f mldi
g mahik
h migajdlbk
i mfdjckgh
j emlkcidha
k mhbljcig
l mjkbhdf
m eagkhiflj

3334555788999

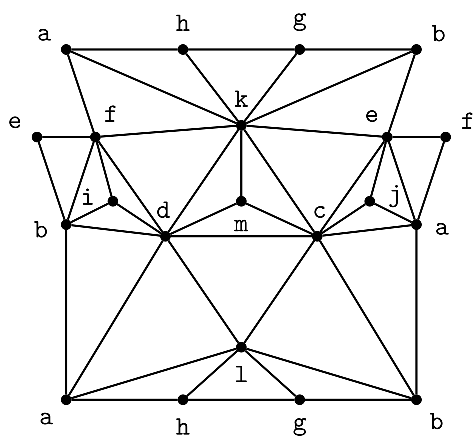
FIGURE 38. tk2k Klein bottle of J. Mayer



a img
b mjh
c igfh
d gehj
e ihdgml
f cgjlmh
g iamedjfc
h icfmbjde
i agchelkjm
j iklfgdhbm
k ilj
l iemfjk
m ijbhflega

(ab)(cd)(ef)(gh)(ij)
3334466688999

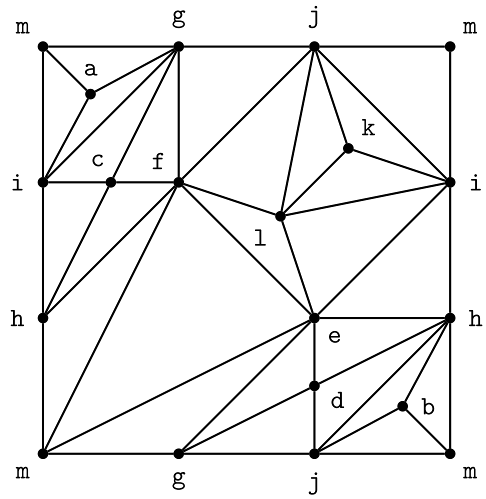
FIGURE 39. tk3t



a bdlhkfejc
b ifekglcad
c bajekmdl
d ibalcmkf
e bfajck
f ibeakd
g blhk
h algk
i bfd
j ace
k bghafdmce
l bcdahg
m dck

(ab)(cd)(ef)(gh)(ij)
3334466688999

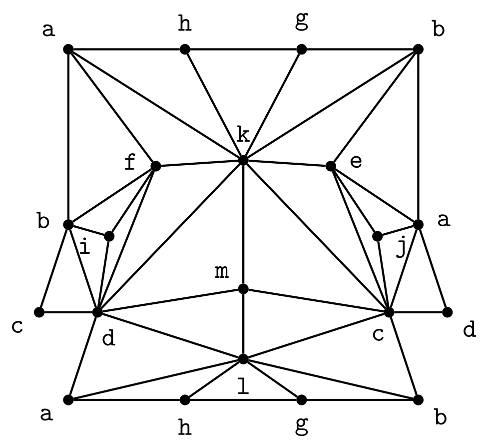
FIGURE 40. tk3k



a img
b mjh
c igfh
d gehj
e ihdgmfl
f cgjlemh
g iamedjfc
h icfmbjde
i chelkjmag
j iklfgdhbm
k ilj
l iefjk
m ijbhfega

(ab)(cd)(ef)(gh)(ij)
3334457788899

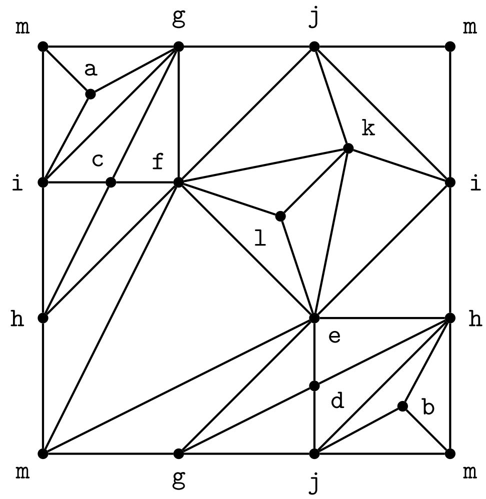
FIGURE 41. tk4t



a bfkhldcje
b ifaekglcd
c bdajekml
d ibcalmkf
e bkcja
f ibakd
g blhk
h lgka
i bfd
j cea
k bghafdmce
l bcmdahg
m dlck

(ab)(cd)(ef)(gh)(ij)
3344455788999

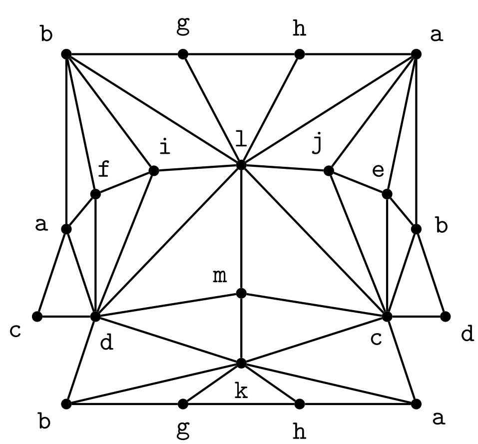
FIGURE 42. tk4k



a gim
b hmj
c higf
d hjge
e hdgmflki
f hcgjklem
g amedjfci
h jdeicfmb
i hekjmagc
j hbmikfgd
k lfjie
l kef
m hfegaijb

(ab)(cd)(ef)(gh)(ij)
3334458888888

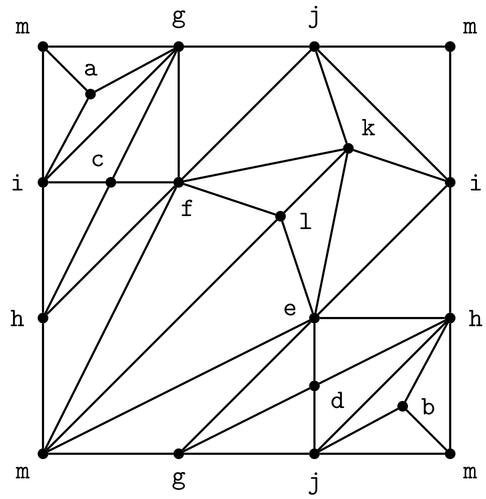
FIGURE 43. tk5t



a hljebfdck
b lgkdceafi
c ljebdakm
d lmkbcafi
e ajcb
f adib
g hlbk
h lgka
i ldfb
j laec
k hgbdmca
l hgbidmcja
m lckd

(ab)(cd)(ef)(gh)(ij)
4444444788999

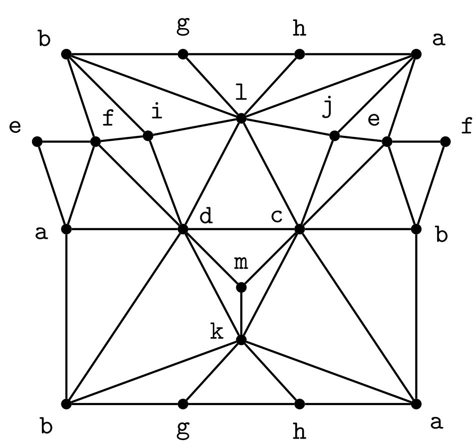
FIGURE 44. tk5k



a mgi
b mjh
c higf
d jgeh
e mlkihdg
f mhcgjkl
g medjfcia
h mbjdeicf
i magchekj
j mikfgdhb
k lfjie
l mfke
m legaijbhf

(ab)(cd)(ef)(gh)(ij)
3344457788889

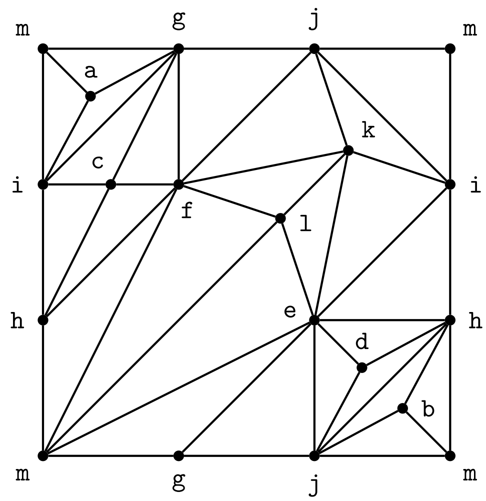
FIGURE 45. tk6t



a ckhljefdb
b cefilgkda
c mkabejld
d mclifabk
e cjafb
f daebi
g kblh
h kalg
i dlbf
j clae
k mcahgbd
l cdibghaj
m ckd

(ab)(cd)(ef)(gh)(ij)
3444455788899

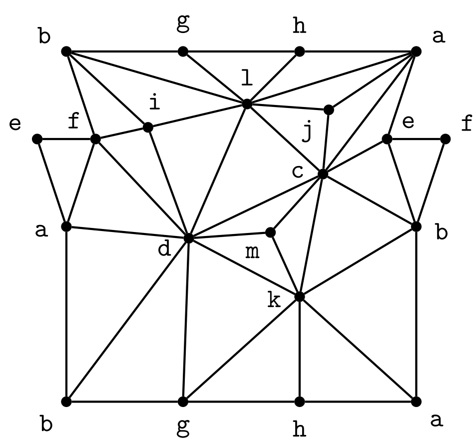
FIGURE 46. tk6k



a gim
b hmj
c fhig
d ehj
e djgmlkih
f cgjklmh
g amejfci
h bjdeicfm
i agchekjm
j bmikfgedh
k elfji
l emfk
m aijbhfleg

3334457788899

FIGURE 47. tk7t



a bdfecjlhk
b adglifeck
c aebkmdlj
d abgkmclif
e acbf
f adibe
g bdkhl
h akgl
i bfdl
j acl
k abcmdgh
l ahgbidcj
m cdk

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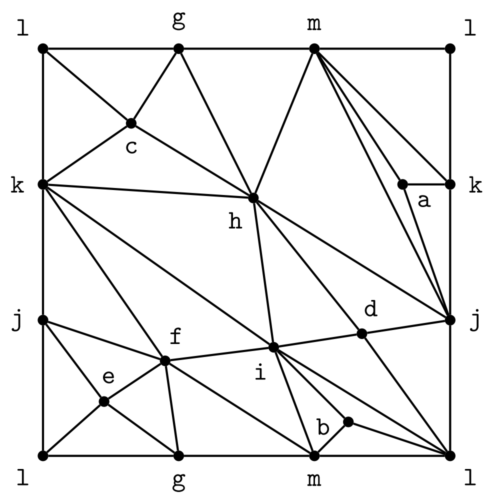
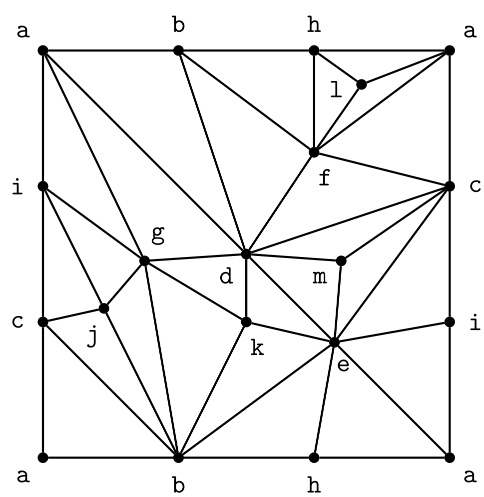
FIGURE 48. tk7k



a mkj
b lmi
c lghk
d lihj
e ljfg
f gejkim
g lefmhc
h cgmjdik
i lbmfkhd
j ldhmakfe
k lchifjam
l ckmbidjeg
m lkajhgfib

3344466778899

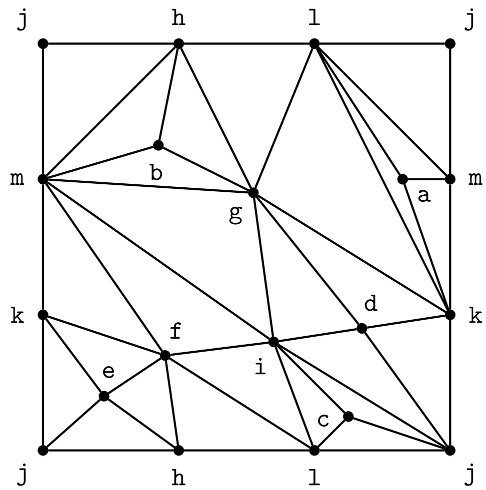
FIGURE 49. tk8t



a lheigdbcf
b acjgkehfd
c afdmeijb
d abfcmekg
e ahbkdmci
f lacdbh
g adkbji
h laebf
i agjce
j cbgi
k bgde
l ahf
m cde

3344556688899

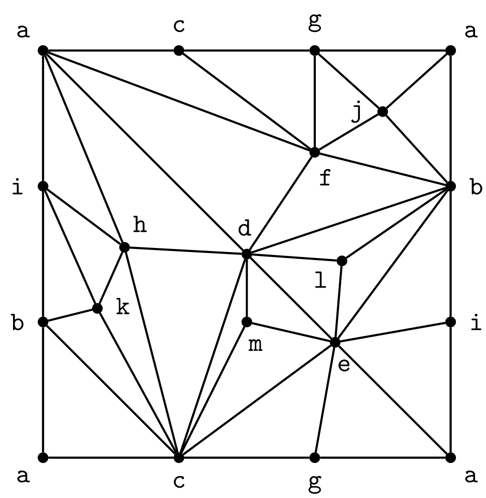
FIGURE 50. tk8k



a lmk
b hgm
c jli
d jigk
e jkfh
f hekmil
g bhlkdim
h jeflgbm
i jclfmgd
j mlcidkeh
k jdglamfe
l jmakghfic
m jhbgifkal

3334467778899

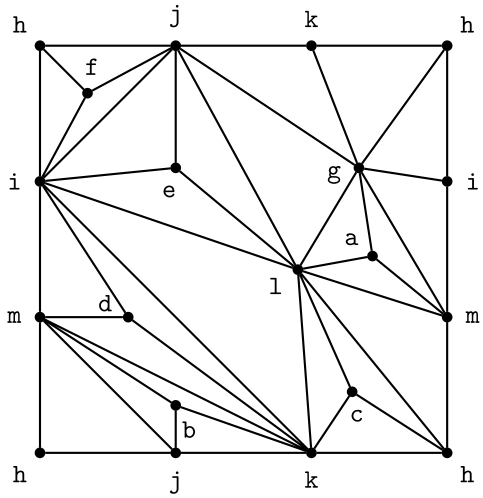
FIGURE 51. tk9t



a jgeihdfcb
b ajfdleikc
c abkhdmegf
d afblemch
e agcmdlbi
f jbdacg
g jaecf
h adcki
i ahkbe
j agfb
k bchi
l bde
m cde

3344555688999

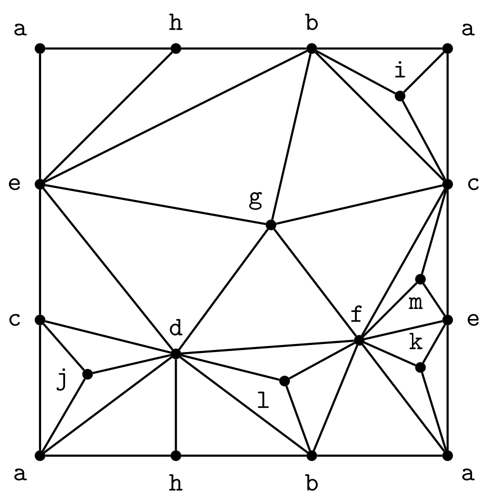
FIGURE 52. tk9k



a lgm
b jmk
c hkl
d ikm
e ijl
f ihj
g imaljkh
h igkclmjf
i jelkdmghf
j ifhmbkgle
k ilchgjbmd
l iejgamhck
m idkbjhlag

3333337899999

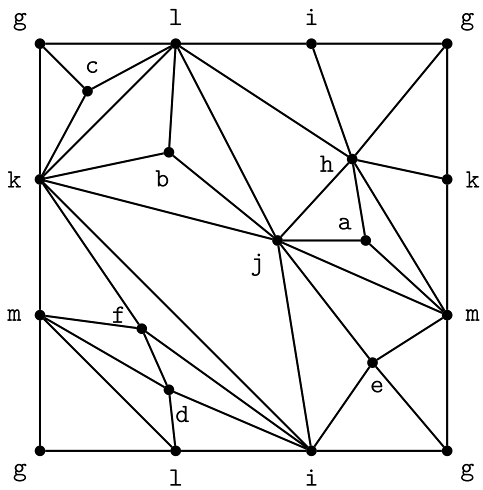
FIGURE 53. tk10t



a ibfkehdjc
b iafldhegc
c iajdemfgb
d ajcegflbh
e ahbgdcmfk
f abldgcmek
g bcfde
h adbe
i abc
j acd
k aef
l bdf
m cef

3333345999999

FIGURE 54. tk10k



a jhm
b klj
c kgl
d lmfi
e gijm
f kidm
g khiemlc
h kmajlig
i kjeghldf
j kblhamei
k lbjifmhgc
l kcgmdihjb
m kfdlgejah

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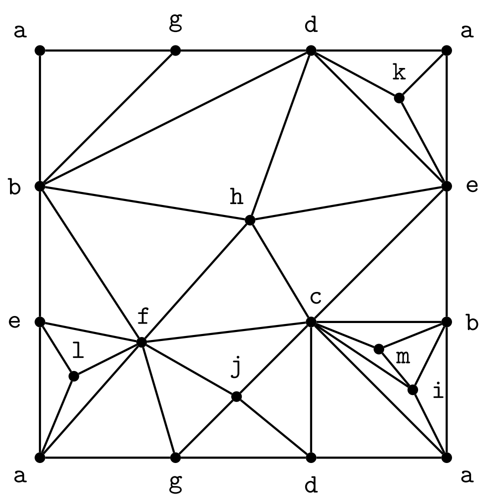
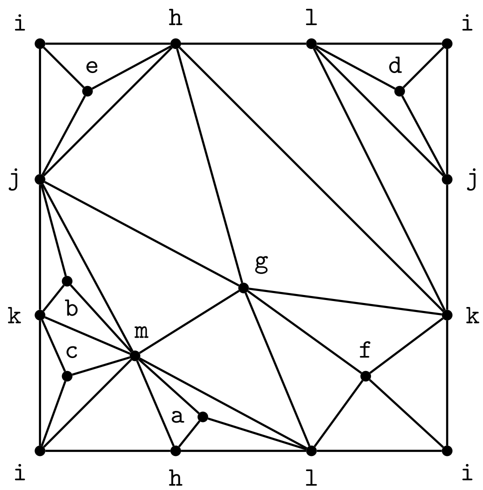
FIGURE 55. tk11t



a kdcibgfle
b agdhfecmi
c adjfhebmi
d kacjgbhe
e kalfbchd
f alebhcjg
g afjdb
h decfb
i abmc
j dgfc
k ade
l aef
m bic

3334455888999

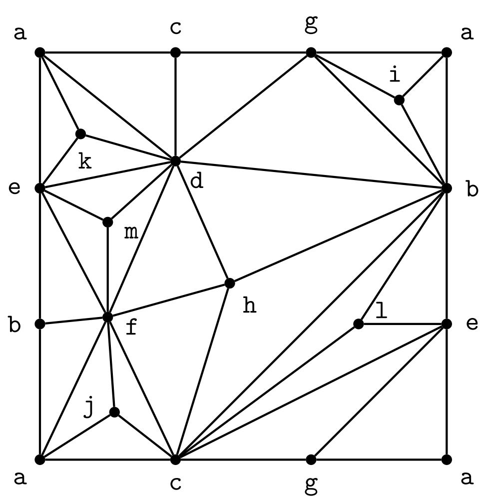
FIGURE 56. tk11k



a hml
b jmk
c ikm
d ijl
e ihj
f ilgk
g flmjhk
h imalkgje
i fkcmhejdl
j iehgmbkld
k ifghljbmc
l idjkhamgf
m ickbjglah

3333346899999

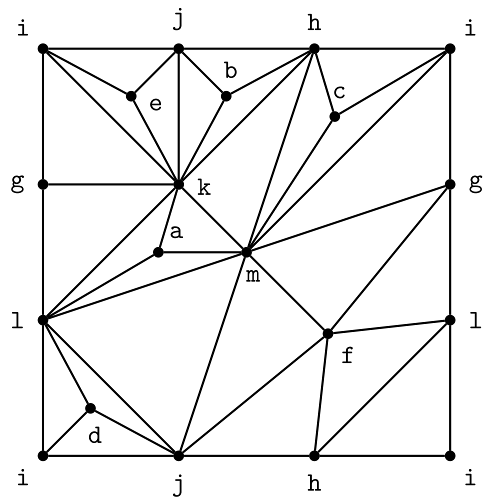
FIGURE 57. tk12t



a igekdcjfb
b iafelchdg
c ajfhblegd
d acgbhfmek
e agclbfmdk
f abemdhcj
g iaecdb
h bcfd
i agb
j afc
k ade
l bce
m fde

3333346899999

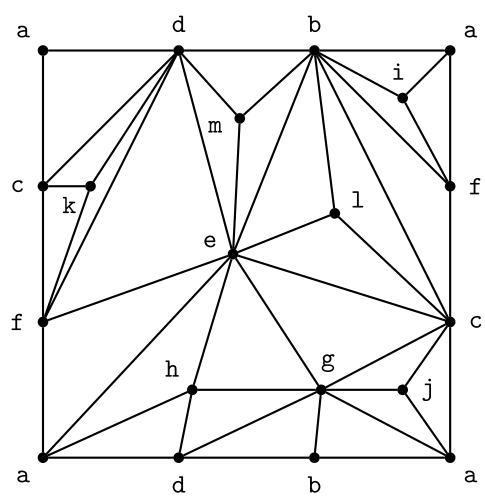
FIGURE 58. tk12k



a kml
b jhk
c imh
d ilj
e ijk
f glhjm
g iklfm
h icmkbjfl
i gmchldjek
j idlmfhbke
k iejbhmalg
l ihfgkamjd
m igfjlakhc

3333355899999

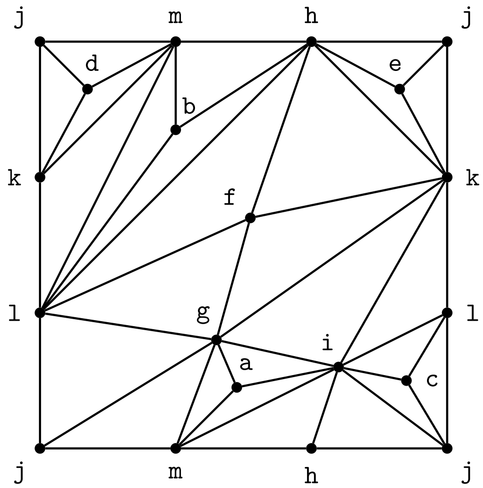
Figure 59. tk13t



a ifehdcjgb
b iagdmelcf
c ajgelbfkd
d ackfembgh
e afdmblcgh
f iaedkcb
g abdhecj
h adge
i afb
j agc
k fcd
l bce
m bde

3333347799999

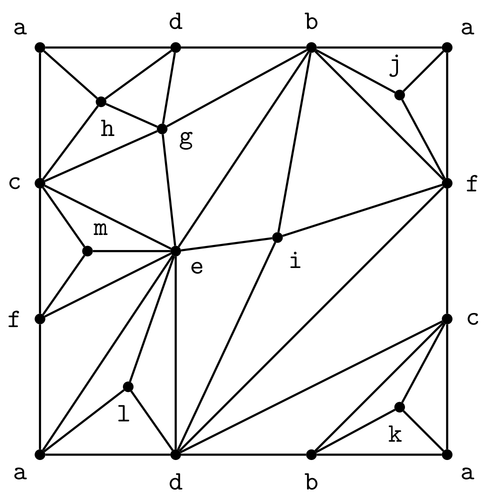
Figure 60. tk13k



a mgi
b hlm
c jil
d jmk
e jkh
f kglh
g jlfkiam
h jekflbmi
i jhmagklc
j kehiclgmd
k jdmligfhe
l jcikmbhfg
m jgaihblkd

3333347889999

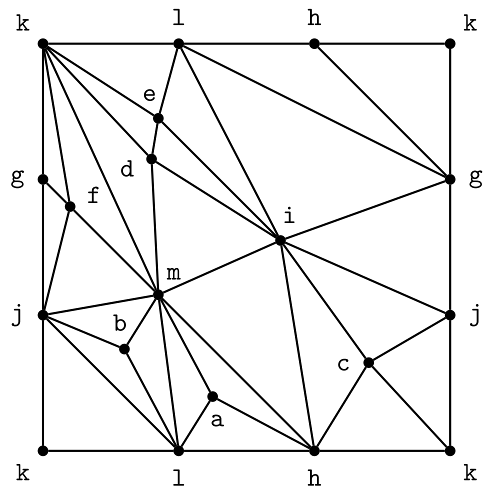
FIGURE 61. tk14t



a jfeldhckb
b jakcdgeif
c akbdfmegh
d ahgbcfiel
e afmcgbidl
f jaemcdib
g bdhce
h acgd
i fbed
j afb
k abc
l ade
m fce

3333445899999

FIGURE 62. tk14k



a hlm
b jml
c khij
d keim
e klid
f kmjg
g kfjilh
h kglamic
i chmdelgj
j kcigfmbl
k cjledmfgh
l kjbmahgie
m kdihalbjf

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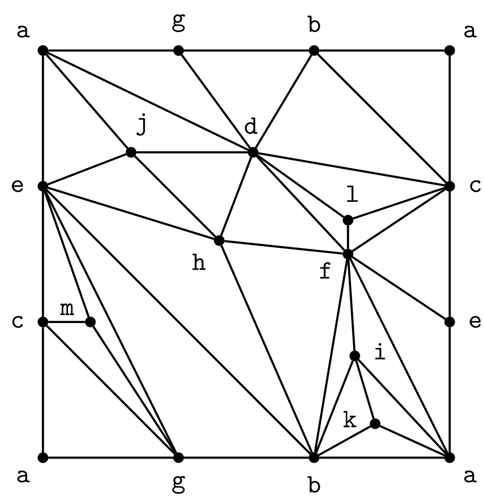
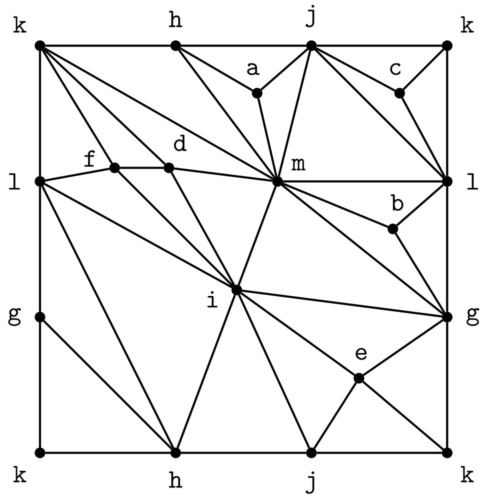
FIGURE 63. tk15t



a kifejdgcb
b kacdgehfi
c abdlfemg
d agbclfhj
e ajhbgmcf
f aibhdlce
g acmebd
h bejdf
i kafb
j adhe
k aib
l cdf
m cge

3334456888899

FIGURE 64. tk15k



a hjm
b lgm
c klj
d kmif
e kjig
f kdil
g keimblh
h kglijam
i dmgejhlf
j kclmahie
k dflcjeghm
l kfihgbmjc
m khajlbgid

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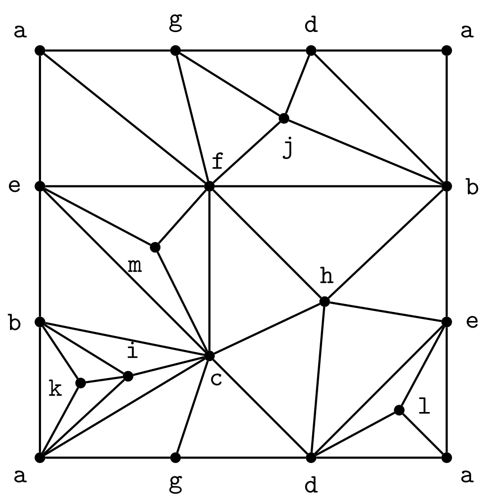
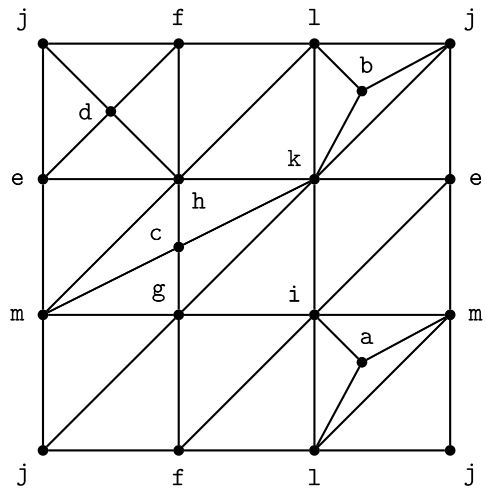
FIGURE 65. tk16t



a kicgfeldb
b kadjfheci
c aibemfhdg
d abjgchel
e aldhbcmf
f aemchbjg
g afjdc
h bedcf
i kacb
j bdgf
k aib
l ade
m efc

3334455888999

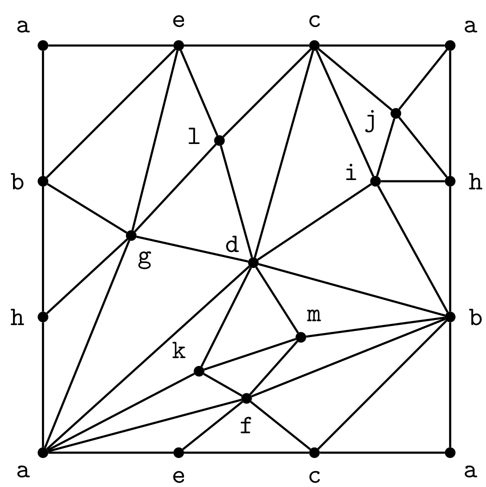
FIGURE 66. tk16k



a iml
b jkl
c hkgm
d jfhe
e jdhmik
f jgilhd
g jmckif
h cmedflk
i alfgkem
j blmgfdek
k jeigchlb
l jbkhfiam
m jlaiehcg

3344666778888

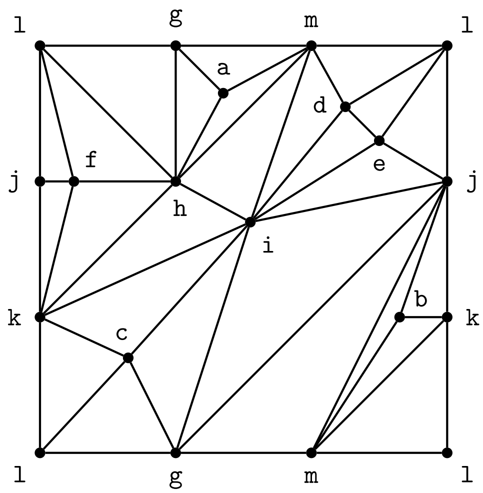
Figure 67. tk17t



a jhgdkfebc
b acfmdihge
c jabfeldi
d akmbiclg
e abglcf
f aecbmk
g ahbeld
h jagbi
i jhbdc
j ahic
k afmd
l cegd
m bfkd

4444556668899

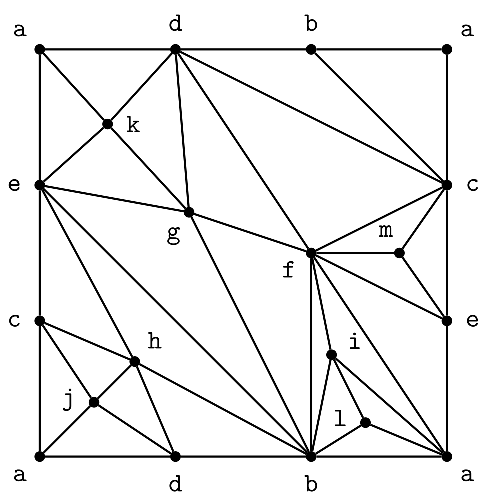
Figure 68. tk17k



a gmh
b jkm
c lkig
d leim
e ljid
f lhkj
g lcijmah
h lgamikf
i ckhmdejg
j lfkbmgie
k lmbjfhic
l cghfjedmk
m ldihagjbk

3344447788899

Figure 69. tk18t



a lifekdjcb
b lacdhegfi
c abdfmehj
d ajhbcfgk
e akgbhcmf
f aibgdcme
g bekdf
h bdjce
i lafb
j achd
k adge
l aib
m cef

3344455888899

Figure 70. tk18k

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